

## Calculus Warm Up # 7-4

## FR practice:

Pick up a sheet on the stool.

For  $t \geq 0$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by

$$v(t) = 1 + 2 \sin\left(\frac{t^2}{2}\right). \text{ The particle is at position } x = 2 \text{ at time } t = 4.$$

- (a) At time  $t = 4$ , is the particle speeding up or slowing down?
- (b) Find all times  $t$  in the interval  $0 < t < 3$  when the particle changes direction. Justify your answer.
- (c) Find the position of the particle at time  $t = 0$ .
- (d) Find the total distance the particle travels from time  $t = 0$  to time  $t = 3$ .

## Classwork: FR practice (tan)

\*No Calculator

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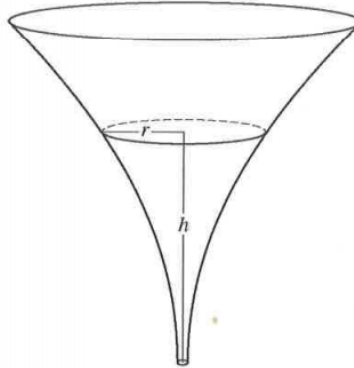
4. Consider the differential equation  $\frac{dy}{dx} = x^2 - \frac{1}{2}y$ .

- (a) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .
- (b) Let  $y = f(x)$  be the particular solution to the given differential equation whose graph passes through the point  $(-2, 8)$ . Does the graph of  $f$  have a relative minimum, a relative maximum, or neither at the point  $(-2, 8)$ ? Justify your answer.
- (c) Let  $y = g(x)$  be the particular solution to the given differential equation with  $g(-1) = 2$ . Find  $\lim_{x \rightarrow -1} \left( \frac{g(x) - 2}{3(x + 1)^2} \right)$ . Show the work that leads to your answer.
- (d) Let  $y = h(x)$  be the particular solution to the given differential equation with  $h(0) = 2$ . Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $h(1)$ .

## backside of the tan FR practice

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\*No Calculator



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height  $h$ , the radius of the funnel is given by  $r = \frac{1}{20}(3 + h^2)$ , where  $0 \leq h \leq 10$ . The units of  $r$  and  $h$  are inches.
- Find the average value of the radius of the funnel.
  - Find the volume of the funnel.
  - The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is  $h = 3$  inches, the radius of the surface of the liquid is decreasing at a rate of  $\frac{1}{5}$  inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

Due FridayClasswork

WU's (2 weeks)

2009 FR

BC - FR Practice  
(purple)HW Quiz by group

10.8, p. 623 (2 days)

10.9, p. 630

12.5, p. 715

HW:

Check answers to purple FR, next slide.

Finish tan FR from today.

# BC - FR Practice (purple) Classwork

Name \_\_\_\_\_

No calculator is allowed for problems on this part of the exam.

1. Consider the function  $f$  given by  $f(x) = xe^{-2x}$  for all  $x \geq 0$ .

(A) Find  $\lim_{x \rightarrow \infty} f(x)$ .

(B) Find the maximum value of  $f$  for  $x \geq 0$ . Justify your answer.

(C) Evaluate  $\int_0^{\infty} f(x) dx$ , or show that the integral diverges.

Points

A)  $\lim_{x \rightarrow \infty} f(x) = 0$

B)  $\frac{1}{2e}$

C)  $\frac{1}{4}$

(A) (1) answer

(2)  $f'(x)$

(B) (1) identify  $x = \frac{1}{2}$   
(1) justification

(C) (2) Antiderivative  
(1)  $\lim_{b \rightarrow \infty}$   
(1) answer

## (purple - backside)

2. The function  $f$  is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n+1)} = 1 + \frac{x-2}{3 \cdot 2} + \frac{(x-2)^2}{3^2 \cdot 3} + \frac{(x-2)^3}{3^3 \cdot 4} + \dots + \frac{(x-2)^n}{3^n(n+1)} + \dots$$

for all real numbers  $x$  for which the series converges.

(A) Determine the interval of convergence of the power series for  $f$ . Show the work that leads to your answer.

(B) Find the value of  $f''(2)$ .

(C) Use the first three nonzero terms of the power series for  $f$  to approximate  $f(1)$ . Use the alternating series error bound to show that this approximation differs from  $f(1)$  by less than  $\frac{1}{100}$ .

A)  $-1 \leq x < 5$

B)  $f''(2) = \frac{2}{27} \rightarrow$  correct answer

C)  $f(1) \approx \frac{47}{54}$ , error  $< \frac{1}{108}$   
 $\frac{1}{108} < \frac{1}{100}$

points:

(1) Approximation

(1) error  $< |4^{\text{th}} \text{ term}|$

(1) analysis

points:  
(A) (1) set up ratio  
(1) find limit of ratio  
(1)  $-1 < x < 5$   
(1) endpoints  
(1) correct answer & analysis